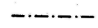
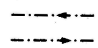
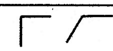
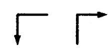
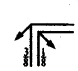
1.4. Graphical symbols for symmetry elements in one, two, and three dimensions

(a) Symmetry planes normal to the plane of projection (three dimensions) and symmetry lines in the plane of the figure (two dimensions)

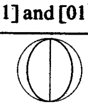
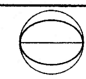
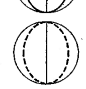
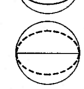
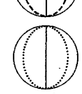
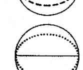
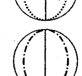
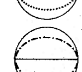
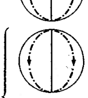
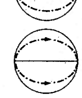
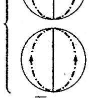
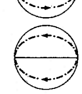
Symmetry plane or symmetry line	Graphical symbol	Glide vector in units of lattice translation vectors parallel and normal to the projection plane	Printed symbol
Reflection plane, mirror plane Reflection line, mirror line (two dimensions)		None	m
'Axial' glide plane Glide line (two dimensions)		$\frac{1}{2}$ along line parallel to projection plane $\frac{1}{2}$ along line in plane	a, b, or c g
'Axial' glide plane		$\frac{1}{2}$ normal to projection plane	a, b, or c
'Diagonal' glide plane		$\frac{1}{2}$ along line parallel to projection plane, combined with $\frac{1}{2}$ normal to projection plane	n
'Diamond' glide plane (pair of planes; in centred cells only)		$\frac{1}{4}$ along line parallel to projection plane, combined with $\frac{1}{4}$ normal to projection plane (arrow indicates direction parallel to the projection plane for which the normal component is positive)	d

(b) Symmetry planes parallel to the plane of projection

Symmetry plane	Graphical symbol‡	Glide vector in units of lattice translation vectors parallel to the projection plane	Printed symbol
Reflection plane, mirror plane		None	m
'Axial' glide plane		$\frac{1}{2}$ in the direction of the arrow	a, b, or c
'Axial' glide plane (in centred cells only)		$\frac{1}{2}$ in either of the directions of the two arrows	a, b, or c
'Diagonal' glide plane		$\frac{1}{2}$ in the direction of the arrow	n
'Diamond' glide plane (pair of planes; in centred cells only)		$\frac{1}{2}$ in the direction of the arrow; the glide vector is always half of a centring vector, *i.e.* one quarter of a diagonal of the conventional face-centred cell	d

‡ Symbols given at upper left corner of the space-group diagrams. A fraction h attached to a symbol indicates two symmetry planes with 'heights' h and $h+\frac{1}{2}$ above the plane of projection; *e.g.* $\frac{1}{8}$ stands for $h=\frac{1}{8}$ and $\frac{5}{8}$. No fraction means $h = 0$ and $\frac{1}{2}$ (*cf.* Section 2.6).

(c) Symmetry planes inclined to the plane of projection (in cubic space groups of classes $\bar{4}3m$ and $m\bar{3}m$ only)

Symmetry plane	Graphical symbol* for planes normal to		Glide vector in units of lattice translation vectors for planes normal to		Printed symbol
	[011] and [01$\bar{1}$]	[101] and [10$\bar{1}$]	[011] and [01$\bar{1}$]	[101] and [10$\bar{1}$]	
Reflection plane, mirror plane			None	None	m
'Axial' glide plane			$\frac{1}{2}$ along [100]	$\frac{1}{2}$ along [010]	a or b
'Axial' glide plane			$\frac{1}{2}$ along [01$\bar{1}$] or along [011]	$\frac{1}{2}$ along [10$\bar{1}$] or along [101]	
'Diagonal' glide plane			$\frac{1}{2}$ along [11$\bar{1}$] or along [$\bar{1}$11]†	$\frac{1}{2}$ along [11$\bar{1}$] or along [$\bar{1}$11]†	n
'Diamond' glide plane (pair of planes; in centred cells only)			$\frac{1}{4}$ along [1$\bar{1}$1] or along [111]‡	$\frac{1}{4}$ along [$\bar{1}$11] or along [111]‡	d
			$\frac{1}{4}$ along [$\bar{1}\bar{1}$1] or along [$\bar{1}$11]‡	$\frac{1}{4}$ along [$\bar{1}\bar{1}$1] or along [1$\bar{1}$1]‡	

* The symbols represent orthographic projections. In the cubic space-group diagrams complete orthographic projections of the symmetry elements around high-symmetry points, such as 0,0,0; $\frac{1}{2}$,0,0; $\frac{1}{4}$,$\frac{1}{4}$,0, are given as 'inserts'.

† In space groups $F\bar{4}3m$ (216), $Fm\bar{3}m$ (225), and $Fd\bar{3}m$ (227), the shortest lattice translation vectors in the glide directions are $\mathbf{t}(1,\frac{1}{2},\bar{\frac{1}{2}})$ or $\mathbf{t}(1,\frac{1}{2},\frac{1}{2})$ and $\mathbf{t}(\frac{1}{2},1,\bar{\frac{1}{2}})$ or $\mathbf{t}(\frac{1}{2},1,\frac{1}{2})$, respectively.

‡ The glide vector is half of a centring vector, *i.e.* one quarter of the diagonal of the conventional body-centred cell in space groups $I\bar{4}3d$ (220) and $Ia\bar{3}d$ (230).

(d) Symmetry axes normal to the plane of projection (three dimensions) and symmetry points in the plane of the figure (two dimensions)

Symmetry axis or symmetry point	Graphical symbol†	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol (subelements in parentheses)
Identity	None	None	1
Twofold rotation axis Twofold rotation point (two dimensions)	⬮	None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Threefold rotation axis Threefold rotation point (two dimensions)	▲	None	3
Threefold screw axis: '3 sub 1'		$\frac{1}{3}$	3_1
Threefold screw axis: '3 sub 2'		$\frac{2}{3}$	3_2
Fourfold rotation axis Fourfold rotation point (two dimensions)	◆ ■	None	4 (2)
Fourfold screw axis: '4 sub 1'		$\frac{1}{4}$	4_1 (2_1)
Fourfold screw axis: '4 sub 2'		$\frac{1}{2}$	4_2 (2)
Fourfold screw axis: '4 sub 3'		$\frac{3}{4}$	4_3 (2_1)
Sixfold rotation axis Sixfold rotation point (two dimensions)	⬢	None	6 (3,2)
Sixfold screw axis: '6 sub 1'		$\frac{1}{6}$	6_1 (3_1,2_1)
Sixfold screw axis: '6 sub 2'		$\frac{1}{3}$	6_2 (3_2,2)
Sixfold screw axis: '6 sub 3'		$\frac{1}{2}$	6_3 (3,2_1)
Sixfold screw axis: '6 sub 4'		$\frac{2}{3}$	6_4 (3_1,2)
Sixfold screw axis: '6 sub 5'		$\frac{5}{6}$	6_5 (3_2,2_1)
Centre of symmetry, inversion centre: '1 bar' Reflection point, mirror point (one dimension)	○	None	$\bar{1}$
Inversion axis: '3 bar'		None	$\bar{3}$ (3,$\bar{1}$)
Inversion axis: '4 bar'		None	$\bar{4}$ (2)
Inversion axis: '6 bar'		None	$\bar{6}$ $\equiv 3/m$
Twofold rotation axis with centre of symmetry		None	$2/m$ ($\bar{1}$)
Twofold screw axis with centre of symmetry		$\frac{1}{2}$	$2_1/m$ ($\bar{1}$)
Fourfold rotation axis with centre of symmetry		None	$4/m$ ($\bar{4}$,2,$\bar{1}$)
'4 sub 2' screw axis with centre of symmetry		$\frac{1}{2}$	$4_2/m$ ($\bar{4}$,2,$\bar{1}$)
Sixfold rotation axis with centre of symmetry		None	$6/m$ ($\bar{6}$,$\bar{3}$,3,2,$\bar{1}$)
'6 sub 3' screw axis with centre of symmetry		$\frac{1}{2}$	$6_3/m$ ($\bar{6}$,$\bar{3}$,3,2_1,$\bar{1}$)

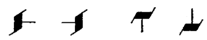
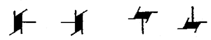
† Notes on the 'heights' *h* of symmetry points $\bar{1}$,$\bar{3}$,$\bar{4}$, and $\bar{6}$:

(1) Centres of symmetry $\bar{1}$ and $\bar{3}$, as well as inversion points $\bar{4}$ and $\bar{6}$ on $\bar{4}$- and $\bar{6}$-axes parallel [001], occur in pairs at 'heights' *h* and $h+\frac{1}{2}$. In the space-group diagrams only one fraction *h* is given, *e.g.* $\frac{1}{4}$ stands for $h=\frac{1}{4}$ and $\frac{3}{4}$. No fraction means $h=0$ and $\frac{1}{2}$. In *cubic* space groups, however, *both* fractions are given for vertical $\bar{4}$-axes, including $h=0$ and $\frac{1}{2}$.

(2) Symmetries $4/m$ and $6/m$ contain vertical $\bar{4}$- and $\bar{6}$-axes; their $\bar{4}$- and $\bar{6}$-inversion points coincide with the centres of symmetry. This is not indicated in the space-group diagrams.

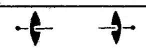
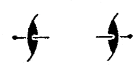
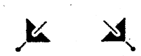
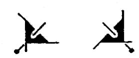
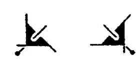
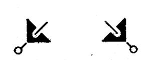
(3) Symmetries $4_2/m$ and $6_3/m$ also contain vertical $\bar{4}$- and $\bar{6}$-axes, but their $\bar{4}$- and $\bar{6}$-inversion points alternate with the centres of symmetry; *i.e.* $\bar{1}$-points at *h* and $h+\frac{1}{2}$ interleave with $\bar{4}$- or $\bar{6}$-points at $h+\frac{1}{4}$ and $h+\frac{3}{4}$. In the tetragonal and hexagonal space-group diagrams only *one* fraction for $\bar{1}$ and *one* for $\bar{4}$ or $\bar{6}$ are given. In the cubic diagrams *all four* fractions are listed for $4_2/m$; *e.g.* $Pm\bar{3}n$ (No. 223): $\bar{1}$: 0,$\frac{1}{2}$; $\bar{4}$: $\frac{1}{4}$,$\frac{3}{4}$.

(e) Symmetry axes parallel to the plane of projection

Symmetry axis	Graphical symbol†		Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol (subelements in parentheses)
Twofold rotation axis			None	2
Twofold screw axis: '2 sub 1'			$\frac{1}{2}$	2_1
Fourfold rotation axis		in cubic space groups only	None	4 (2)
Fourfold screw axis: '4 sub 1'		in cubic space groups only	$\frac{1}{4}$	$4_1\,(2_1)$
Fourfold screw axis: '4 sub 2'		in cubic space groups only	$\frac{1}{2}$	$4_2\,(2)$
Fourfold screw axis: '4 sub 3'		in cubic space groups only	$\frac{3}{4}$	$4_3\,(2_1)$
Inversion axis: '4 bar'		in cubic space groups only	None	$\bar{4}$ (2)
Inversion point on '4 bar'-axis		in cubic space groups only		$\bar{4}$-point

† Symbols for horizontal symmetry axes are given outside the unit cell of the space-group diagrams. *Twofold* axes always occur in pairs, at 'heights' h and $h + \frac{1}{2}$ above the plane of projection; here, a fraction h attached to such a symbol indicates two axes with heights h and $h + \frac{1}{2}$. No fraction stands for $h = 0$ and $\frac{1}{2}$. The rule of pairwise occurrence is not valid for the horizontal *fourfold* axes in cubic space groups; here, *all* heights are given, including $h = 0$ and $\frac{1}{2}$. This applies also to the horizontal $\bar{4}$-axes and the $\bar{4}$-inversion points located on these axes.

(f) Symmetry axes inclined to the plane of projection (in cubic space groups only)

Symmetry axis	Graphical symbol†		Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol
Twofold rotation axis		parallel to a face diagonal of the cube	None	2
Twofold screw axis: '2 sub 1'		parallel to a face diagonal of the cube	$\frac{1}{2}$	2_1
Threefold rotation axis		parallel to a body diagonal of the cube	None	3
Threefold screw axis: '3 sub 1'		parallel to a body diagonal of the cube	$\frac{1}{3}$	3_1
Threefold screw axis: '3 sub 2'		parallel to a body diagonal of the cube	$\frac{2}{3}$	3_2
Inversion axis: '3 bar'		parallel to a body diagonal of the cube	None	$\bar{3}$

† Dots mark intersection points of axes with the plane at $h = 0$. In some cases the intersection points are obscured by symbols of symmetry elements with height $h \geq 0$; examples: $Fd\bar{3}$ (203), origin choice 2; $Pn\bar{3}n$ (222), origin choice 2; $Pm\bar{3}n$ (223); $Im\bar{3}m$ (229); $Ia\bar{3}d$ (230).

Notes on graphical symbols of symmetry elements

(i) The graphical symbols and their explanations (columns 2 and 3) are independent of the projection direction and the choice of the basis vectors. They are, therefore, applicable to any projection diagram of a space group. The printed symbols of *glide planes* (column 4), however, may change with a change of the basis vectors.

In the rhombohedral space groups $R3c$ and $R\bar{3}c$, for instance, the dotted line refers to a c glide when described with 'hexagonal axes' and projected along [001]; for a description with 'rhombohedral coordinates' and projection along [111], the same glide plane would be called n. The parallel n glide in the hexagonal description becomes an a, b, or c glide in the rhombohedral description; *cf* Section 1.3.

(ii) The graphical symbols for glide planes in column 2 are not only used for the glide planes defined in Section 1.3, but also for the further glide planes g which are mentioned in Section 1.3 and explained in Sections 2.5 and 11.2.

(iii) In monoclinic space groups the 'parallel' glide vector of a glide plane may be along a lattice translation vector which is inclined to the projection plane.